6-4: APPLICATIONS OF LINEAR SYSTEMS

Lesson Objectives:

- *Understand, write, and solve systems of linear equations from word problems*

1 Writing Systems of Linear Equations

EXAMPLE 1: MIXTURE PROBLEMS

1. To produce a 50% mixture of antifreeze, a 30% mixture of antifreeze is mixed with pure antifreeze. How much of each type is needed to produce 7 quarts of the 50% mixture?

100%

Let x = qts of 30% antifreeze
y = qts of 100% antifreeze

$$\begin{cases} x + y = 7 \\ .3x + y = .5(7) \Rightarrow 3.5 \end{cases}$$

$-.3x - y = -3.5$

$.7x = 3.5$, $\frac{.7x}{.7} = \frac{3.5}{.7}$, $\boxed{x = 5}$

$5 + y = 7$, $-5 \quad -5$, $\boxed{y = 2}$

5 qts of 30% antifreeze
2 qt of pure antifreeze

2. A metalworker has some ingots of metal alloy that are 20% copper and others that are 60% copper. How many kilograms of each type of ingot should the metalworker combine to create 80 kg of a 52% copper alloy?

Let x = kg of 20% copper
y = kg of 60% copper

$$-.2\begin{cases} x + y = 80 \\ .2x + .6y = .52(80) \end{cases} \Rightarrow \begin{array}{r} -.2x - .2y = -16 \\ + \; .2x + .6y = 41.6 \end{array}$$

$\frac{.4y}{.4} = \frac{25.6}{.4}$, $\boxed{y = 64}$

$x + 64 = 80$, $-64 \quad -64$, $\boxed{x = 16}$

16 kg of 20% copper
64 kg of 60% copper

3. Suppose you combine ingots of 25% copper alloy and 50% copper alloy to create 40 kg of 45% copper alloy. How man kilograms of each do you need?

Let x = kg of 25% copper
y = kg of 50% copper

$$-.25\begin{cases} x + y = 40 \\ .25x + .5y = .45(40) \end{cases} \Rightarrow \begin{array}{r} -.25x - .25y = -10 \\ + \; .25x + .5y = 18 \end{array}$$

$\frac{.25y}{.25} = \frac{8}{.25}$, $\boxed{y = 32}$

$x + 32 = 40$, $-32 \quad -32$, $\boxed{x = 8}$

8 kg of 25% copper
32 kg of 50% copper

4. A 20 oz alloy of platinum that costs \$220 per ounce is mixed with an alloy that costs \$400 per ounce. How many ounces of the \$400 alloy should be used to make an alloy that costs \$300 per ounce?

Let y = oz of \$400 alloy
z = oz of \$300 alloy

$$\begin{cases} 20 + y = z \\ 220(20) + 400y = 300z \end{cases}$$

$220(20) + 400y = 300(20 + y)$
$4400 + 400y = 6000 + 300y$
$-300y \quad -300y$
$4400 + 100y = 6000$
$-4400 \quad -4400$
$\frac{100y}{100} = \frac{1600}{100}$
$\boxed{y = 16}$

16 oz of \$400/oz alloy

Always $x = $ #items or services

$y = \$$ amount

EXPENSES : $y =$

INCOME : $y =$

EXAMPLE 2: FINDING A BREAK-EVEN POINT

rate

5. Suppose a model airplane club publishes a newsletter. Expenses are $0.90 for printing and mailing each copy, plus $600 (initial) total for research and writing. The price of the newsletter is $1.50 per copy. How many copies of the newsletter must the club sell to break even?

Let $x =$ newsletters

$y = \$$ amount

EXPENSES: $y = 0.9x + 600$

INCOME: $y = 1.5x$

$0.9x + 600 = 1.5x$

$-0.9x \quad -0.9x$

$\frac{600}{0.6} = \frac{0.6x}{0.6}$

$\boxed{1000 = x}$

1000 newsletters

6. Suppose you are starting an office-cleaning service. You have spent $315 on equipment. To clean an office, you use $4 worth of supplies. You charge $25 per office. How many offices must you clean to break even?

Let $x =$ offices

$y = \$$ amount

$y = 315 + 4x$

$y = 25x$

$315 + 4x = 25x$

$-4x \quad -4x$

$\frac{315}{21} = \frac{21x}{21}$

$\boxed{15 = x}$

15 offices

7. Suppose you invest $1500 in equipment to put pictures on T-shirts. You buy each T-shirt for $3. After you have placed the picture on the shirt, you sell it for $20. How many T-shirts must you sell to break even?

Let $x =$ t-shirts

$y = \$$ amount

$y = 1500 + 3x$

$y = 20x$

$1500 + 3x = 20x$

$-3x \quad -3x$

$\frac{1500}{17} = \frac{17x}{17}$

$88.23\ldots = x$

89 t-shirts

EXAMPLE 3: AIRSPEED/WINDSPEED PROBLEMS

$R \cdot T = D$

$(A + w) \cdot T = D$

$(A - w) \cdot T = D$

8. Suppose you fly from Miami, Florida to San Francisco, California. It takes 6.5 hours to fly 2600 miles against a head wind. At the same time, your friend flies from San Francisco to Miami. Her plane travels at the same average airspeed, but her flight only takes 5.2 hours. Find the average wind speed.

Let $A =$ airspeed

$w =$ windspeed

$\frac{(A+w) \cdot \cancel{5.2}}{\cancel{5.2}} = \frac{2600}{5.2}$

$\frac{(A-w) \cdot \cancel{6.5}}{\cancel{6.5}} = \frac{2600}{6.5}$

$A + w = 500$

$A - w = 400$

$+ \; A + w = 500$

$\frac{2A}{2} = \frac{900}{2}$

$\boxed{A = 450}$

$450 + w = 500$

$-450 \quad -450$

$\boxed{w = 50}$

Plane : 450 mph

Wind : 50 mph

9. A boat travels 60 km upstream (against the current) in 5 hours. The boat travels the same distance downstream in 3 hours. What is the rate of the boat in still water? What is the rate of the current?

Let B = boatspeed
C = currentspeed

$\frac{(B+C)\cancel{3}}{\cancel{3}} = \frac{60}{3}$ $\frac{(B-C)\cancel{5}}{\cancel{5}} = \frac{60}{5}$

$B+C=20$ $B-C=12$

$+\ B+C=20$

$\frac{2B}{2} = \frac{32}{2}$

$\boxed{B=16}$

$16+C=20$
$-16 \quad -16$
$\boxed{C=4}$

Boat: 16 km/h
Current: 4 km/h

10. When Lucy swims with the current, she swims 18 km in 2 hours. Against the current, she can swim only 14 km in the same time. How fast can Lucy swim in still water? What is the rate of the current?

Let L = Lucy's speed
C = current speed

$\frac{(L+C)\cdot\cancel{2}}{\cancel{2}} = \frac{18}{2}$ $\frac{(L-C)\cdot\cancel{2}}{\cancel{2}} = \frac{14}{2}$

$L+C=9$ $L-C=7$

$+\ L+C=9$

$\frac{2L}{2} = \frac{16}{2}$

$\boxed{L=8}$

$8+C=9$
$-8 \quad -8$
$\boxed{C=1}$

Lucy: 8 km/h
Current: 1 km/h

11. On an upstream trip, a canoe travels 40 km in 5 hours. Downstream, it travels the same distance in half the time. What is the rate of the canoe in still water and the rate of the current?

Let B = canoe's speed
C = current speed

$\frac{(B+C)\cancel{2.5}}{\cancel{2.5}} = \frac{40}{2.5}$ $\frac{(B-C)\cancel{5}}{\cancel{5}} = \frac{40}{5}$

$B+C=16$ $B-C=8$

$+\ B+C=16$

$\frac{2B}{2} = \frac{24}{2}$

$\boxed{B=12}$

$12+C=16$
$-12 \quad -12$
$\boxed{C=4}$

Canoe: 12 km/h
Current: 4 km/h

12. A duck can fly 2400 m in 10 min with the wind. Against the wind, it can fly only two thirds of this distance in 10 min. How fast could the duck fly in still air? What is the rate of the wind?

Let D = duck's speed
W = windspeed

$\frac{(D+W)10}{10} = \frac{2400}{10}$ $\frac{(D-W)10}{10} = \frac{1600}{10}$

$D+W=240$ $D-W=160$

$+\ D+W=240$

$\frac{2D}{2} = \frac{400}{2}$

$\boxed{D=200}$

$200+W=240$
$-200 \quad -200$
$\boxed{W=40}$

Duck: 200 m/min
Wind: 40 m/min

3 to 2

$3:2$

$\frac{3}{2}$

EXAMPLE 4: RATIO PROBLEMS

13. Kay spends 250 min/week exercising. Her ratio of time spent on aerobics to time spent weight training is 3 to 2. How many minutes per week does she spend on aerobics? How many minutes per week does she spend on weight training?

Let x = time on aerobics

y = time on weights

150 min/wk on aerobics

100 min/wk on weights

$$\begin{cases} x+y=250 \\ \cancel{y}\left(\frac{x}{\cancel{y}}\right)=\left(\frac{3}{2}\right)y \end{cases}$$

$x=\frac{3}{2}y$

$\frac{3}{2}y+\frac{2}{2}y=250$

$\frac{\cancel{2}}{\cancel{5}}\left(\frac{\cancel{5}}{\cancel{2}}y\right)=(250)\frac{2}{5}$

$\boxed{y=100}$

$x+100=250$
$-100 \quad -100$

$\boxed{x=150}$

14. Suppose the ratio of girls to boys in your school is 19:17. There are 1908 students altogether. Write and solve a system of equations to find the total number of boys and girls in the school.

Let b = boys

g = girls

901 boys

1007 girls

$b+g=1908$

$\cancel{b}\left(\frac{g}{\cancel{b}}\right)=\left(\frac{19}{17}\right)b$

$g=\frac{19}{17}b$

$\frac{17}{17}b+\frac{19}{17}b=1908$

$\frac{\cancel{17}}{\cancel{36}}\left(\frac{\cancel{36}}{\cancel{17}}b\right)=(1908)\frac{17}{36}$

$\boxed{b=901}$

$901+g=1908$
$-901 \quad -901$

$\boxed{g=1007}$

EXAMPLE 5: TWO NUMBERS PROBLEMS

15. Three times the larger of two numbers is equal to four times the smaller. The sum of the numbers is 21. Find the numbers.

Let x = larger #

y = smaller #

12 & 9

$$\begin{cases} 3\cdot x=4\cdot y \\ x+y=21 \end{cases}$$
$-y \quad -y$

$x=21-y$

$x=21-9$

$\boxed{x=12}$

$3(21-y)=4y$

$63-3y=4y$
$+3y \quad +3y$

$\frac{63}{7}=\frac{7y}{7}$

$\boxed{9=y}$

16. The difference between two numbers is 16. Five times the smaller is the same as 8 less than twice the larger. Find the numbers.

Let x = larger #

y = smaller #

24 & 8

$+y \quad +y$
$x-y=16$

$5y=2x-8$

$x=16+y$

$x=16+8$

$\boxed{x=24}$

$5y=2(16+y)-8$

$5y=32+2y-8$

$5y=24+2y$
$-2y \quad -2y$

$\frac{3y}{3}=\frac{24}{3}$

$\boxed{y=8}$

Name ___________________________ ***6-4 Practice Worksheet*** Period __________

Write and solve a system of equations for each problem.

1. A 4% solution of borax must be added to a 12% solution of borax to obtain 32 pounds of a 10% solution of borax. How many pounds of each type of solution should be used?

2. One mixture of salt water is 10% salt. Another mixture is 3% salt. How many pounds of salt water of each type must be mixed to produce 42 pounds of 5% salt water?

3. 50 grams of a solution which is 36% acid is needed by mixing together a 20% acid solution with pure (100%) acid. How many grams of each type should be used?

4. When a plane flies into the wind, it can travel 3000 km in 6 hours. When it flies with the wind, it can travel the same distance in 5 hours. Find the rate of the plane in still air and the rate of the wind.

5. With the wind, a jet can fly 2500 km in 2 hours and 30 minutes. Against the wind, it can fly only 2000 km in the same amount of time. Find the rate of the jet in still air and the rate of the wind.

6. With the wind, a plane flew 1400 km in 4 hours. On the return trip, the pilot was forced to land after 1 hour and 30 minutes, having traveled only 450 km. Find the rate of the plane in still air and the rate of the wind.

7. A salmon swims 100 m in 8 min downstream. Upstream, it would take the fish 20 min to swim the same distance. What is the rate of the salmon in still water? What is the rate of the current?

8. A new parking lot has spaces for 450 cars. The ratio of spaces for full-sized cars to compact cars is 11 to 4. How many spaces are for full-sized cars? How many spaces are for compact cars?

9. The larger of two numbers is 1 more than twice the smaller. The sum of the numbers is 20 less than three times the larger. Find the numbers.

10. The sum of two numbers is the same as four times the smaller number. If twice the larger is decreased by the smaller, the result is 30. Find the numbers

11. You start a company that sells widgets online. The initial equipment and set up cost $1200. It costs an additional $10 for every widget produced. If the widgets are sold for $35, how many must be sold to break even?

12. The freshman class is selling t-shirts as a fundraiser. They found a company that will charge $125 to make the printing screen plus $8.75 per shirt. If the class sells the shirts for $15 each, how many must they sell to break even?

13. The price of a sweater is $5 less than twice the price of a shirt. If four sweaters and three shirts cost $200, find the price of each shirt and each sweater.

14. A shipment of TV sets, some weighing 30 pounds each and others weighing 50 pounds each has a total weight of 880 pounds. If there are 20 TV sets altogether, how many weigh 50 pounds?

15. Your teacher is giving you a test worth 100 points containing 40 questions. There are two-point questions and four-point questions on the test. How many of each type of question are on the test?